OSCILLATOR PHASE NOISE OPTIMIZATION AND CORRECTION

Maxim Goryachev, Serge Galliou, Philippe Abbé

Institut FEMTO-ST (UMR CNRS 6174), Département Temps Fréquence
ENSMM 26, Chemin de l'Epitaphe, 25000 Besançon, France
Maxim.Goryachev@femto-st.fr, Serge.Galliou@ens2m.fr, Philippe.Abbe@ens2m.fr

INTRODUCTION

Nowadays electronical engineers widely use different computer-aided design (CAD) tools for electronic circuit modelling and optimization. These tools can help significantly to speed up the process of design and to increase accuracy, but sometimes they meet the limits where these advantages are lost. A typical example is the modelling of electronical oscillators based on quartz crystal resonators with outstanding high quality (Q) factor. The authors have faced the stated problem within the framework of the development of a cryogenic bulk-acoustic-wave (BAW) crystal oscillator at a relatively low frequency (below 50 MHz). At usual temperatures, ultra stable oscillators (whose fractional frequency stability is better than $1\cdot 10^{-13}$ over measuring times of a few seconds) are based on high-quality quartz crystal resonators exhibiting quality factors typically close to $Q = 1.2\cdot 10^{6}$ at a resonance frequency $f = 10$ MHz: their $Q\cdot f$ product is rather constant for a given quartz crystal cut, working on a given BAW resonance mode, i.e. $Q\cdot f \approx 12\cdot 10^{12}$. On the other hand, when such a resonator is activated at a temperature lower than 10 K, its $Q\cdot f$ product increases dramatically and can reach $3800\cdot 10^{12}$ at 4.2K [1], [2]. Also, SiGe heterojunction bipolar transistor are proved to be a good choice of an low-noise active device at cryogenic temperatures (details of device measurements and modelling can be found in [8]). So, such low-noise active devices and high-Q resonators provide a possibility of the cryogenic oscillator creation.

Usually two common solutions of quartz oscillator modelling are used. The first solution is a combination of AC and Transient analyses with ordinary SPICE simulators. The AC analysis is used to find the oscillation frequency and the in-circuit quality factor. The transient analysis is used to check those results, but the circuit Q is significantly reduced in order to decrease simulation time and often to get an algorithm convergence towards a nontrivial solution. As a result it is not possible to obtained true values of currents and voltages in the circuit (as well as such an important parameter as the quartz crystal dissipation power). In addition to this, optimization of the system may pose additional problems, since it requires both AC and Transient analyses on each iteration step. So, this process can hardly be automated.

The second solution is known as harmonic balance (HB) technique. This type of analysis is realized in some commercial RF-circuit simulators, such as ADS from Agilent and Harmonica (Serenade) from Ansof. Nevertheless, such sophisticated and expensive simulators are capable of high-Q oscillator modelling, but this task is very time consuming and requires very good initial guess for oscillation frequency (since there is now explicit source of frequency in the circuit). The last point requires an additional stage of AC Analysis on each iteration step.

Whatever the case, for extroardinary values of quality factor which can be reached at cryogenic temperatures convergence to the result is not guaranteed. These facts make automatic optimization difficult and inefficient.

To overcome all the difficulties connected to high-Q oscillators modelling we propose another simple but effective technique, which is more analytical than numerical and does not require any SPICE or Harmonic Balance Simulators. In rather simple circuits all the concerning parameters may be calculated analytically or, in more elaborated cases, realized with simple home-made software. These parameters include the oscillating frequency, the amplitude of the first harmonic and DC solution, the active power dissipated by the resonator and the oscillator, the in-circuit Q-factor.

In the fillowing sections we will discuss different stages of the proposed technique. For the explanation a Colpitss quartz crystal oscillator (see Fig. 1) is used, but this technique can be used for other types of oscillators with lumped components.

OSCILLATING FREQUENCY AND FIRST HARMONIC AMPLITUDE CALCULATION

At this stage of the present technique one should transform an oscillator circuit into a closed loop system with linear and nonlinear elements, which is widely used in control system theory. The main idea here is to unite all linear dynamic components (inductors, capacitors and resistances), and process them as one linear block, which is connected in parallel with a nonlinear one. In our case the nonlinear component is a BJT or HBT transistor. The quartz crystal is considered here as a linear network, which consists of a series motional branch (elements C_x, R_x and L_x) and parallel capacitance C_0.

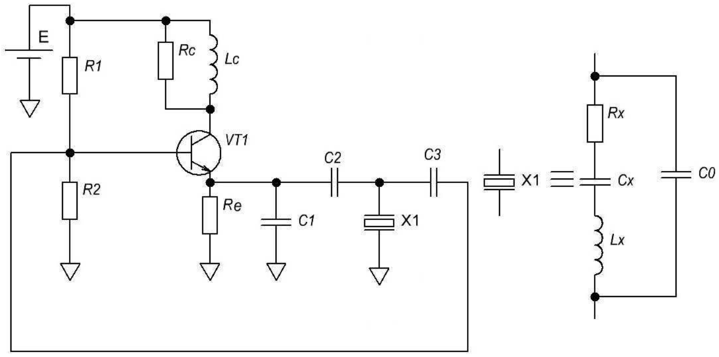

Fig. 1. Colpitts quartz crystal oscillator

From the oscillator circuit which is shown in Fig. 1 one may obtain its full equivalent circuit shown in Fig. 2 (a). To do this it is needed to apply not only the rules of parallel and series connection of elements, but also Y-Δ equivalent transforms. As a result, the oscillator consists of linear elements $Z_1(p)$, $Z_2(p)$, $Z_3(p)$, $Z_4(p)$ and $Z_5(p)$, where $p \equiv \frac{d}{dt}$.

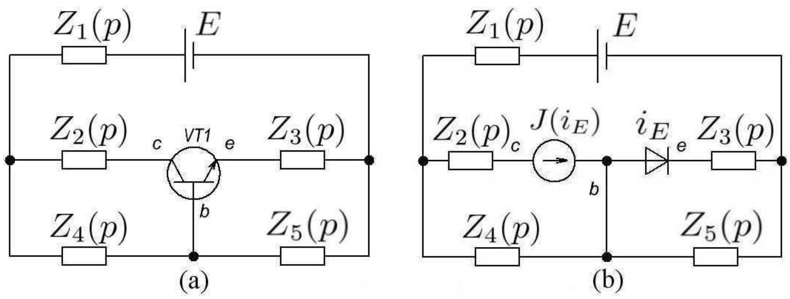

Fig. 2. Steps of oscillator transformations

A nonlinear model of the BJT transistor must be considered, here an Ebers-Moll model (see Fig. 2 (b)) consisting of a nonlinear element $i_E = I_f(u_{BE})$ and a linear current source $J = \alpha_f i_E$.

Now, following the same rules of linear elements transformation one may obtain the one loop oscillating circuit with one source of bias (see Fig. 3 (a)), one linear and one nonlinear blocks. The dependent current source is transformed into a dependent voltage source and then into negative resistance. Finally, the system dynamics can be represented with a single formula:

$$E \cdot H_z(p) = u_{BE} + Z_l(p) \cdot I_f(u_{BE}), \tag{1}$$

where u_{BE} is the system variable (voltage between base and emitter of the transistor). The simple circuit shown in Fig. 3 (a) characterized with equation (1) may be considered as a closed loop in the nonlinear control theory (see Fig. 3 (a)). Such a loop has two main variables: the emitter current i_E and the voltage base-emitter u_{BE}.

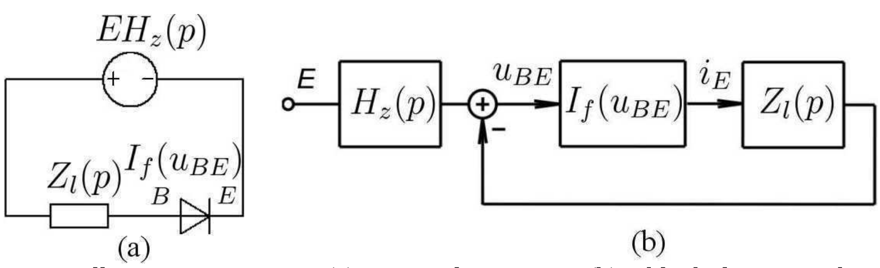

Fig. 3. One loop oscillator representation: (a) - equivalent circuit, (b) - block-diagram, where $I_f(u_{BE})$ is a nonlinear part

The resulting circuit (Fig. 3) has the same dynamics as the initial one (Fig. 1), since no simplification (except for the model of the transistor) has been made. This means that this loop can also be used for the oscillator modelling as well as the initial circuit. The fundamental difference between these two variants is that the former represents the lowest order of entropy, and the latter is its variant with the highest entropy level (so it may correspond to a vast class of lumped oscillators).

The oscillator behaviour can be regarded from the point of view of the block-diagram Fig. 3 (b), so that all methods of nonlinear control system investigation from the control theory (including investigation of oscillation conditions) can be used here. Different methods have already been investigated to find amplitude and frequency of oscillations for this type of nonlinear systems. In this work we consider the method of describing functions. The describing function method (which is also sometimes called the Harmonic Balance method in control theory) does not have a strict mathematical proof, though it is physically clear and correct. This method is very usefull for high-order systems and helps to find oscillation conditions. The main criterion of the method is so-called "filter hypothesis", which means that the linear part of the system has to filter the main harmonic and suppress others which arise at the output of the nonlinear element. This can be quantitatively expressed as follows:

$$\left|\frac{i_{Ek}}{i_{E1}}\right|\frac{Z_l(jk\omega)}{Z_l(j\omega)} \ll 1, \tag{2}$$

where i_{E1} and i_{Ek} are correspondingly amplitudes of the first and kth harmonics of the emitter current ($k \geqslant 2$). It is clear that this condition is satisfied for such transfer functions which have helical gain-phase characteristic. Thus, in oscillating regime a harmonic balance is reached: high-order harmonics of emitter current produced by nonlinear part are suppressed by linear one, and the signal u_{BE} (which is closed to a sine signal, since harmonics are suppressed) is applied to the nonlinear part.

To illustrate the method a following example is considered: Fig. 4 shows Bode plot of oscillator linear parts ($Z_l(j\omega)$) for different values of quartz quality factor: 10^5, 10^6, 10^7 for the same values of resonance frequency and motional resistance.

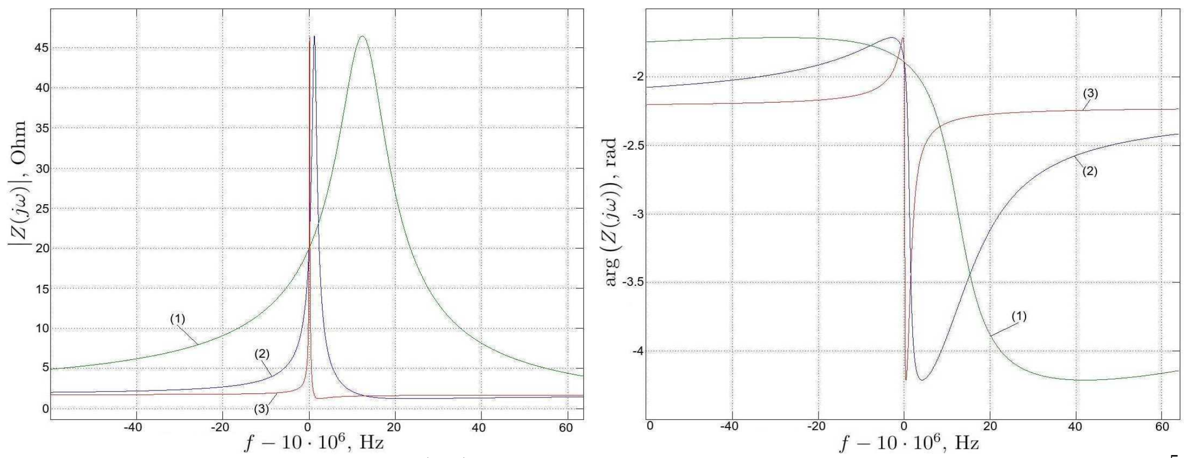

Fig. 4. Bode plot of the linear part $Z_l(j\omega)$ for different values of the quartz crystal quality factor Q: (1) - 10^5, (2) - 10^6, (3) - 10^7 (with the same values of resonance frequency and motional resistance). Numerical values (see Fig. 1) are $E = 5$V, $R_c = 330\Omega$, $L_c = 10$uH, $R_1 = 10$kΩ, $R_2 = 15$kΩ, $R_e = 1.5$kΩ, $C_1 = 150$pF, $C_2 = 330$pF, $C_3 = 1$nF, $C_4 = 180$pF, $\alpha_f = 0.99$, $r_0 = 1\Omega$. The quartz crystal has the resonant frequency of $f_r = 10$MHz, motinal resistance of $R_x = 94\Omega$ and shunt capacitance $C_0 = 2$pF.

From Fig. 4 it is clear that the linear part of the loop works as a selective filter with a $-\pi$ phase shift at the oscillating frequency (another $-\pi$ phase shift to fulfill the Barkhausen conditions originates from the negative sign of the feedback). So, the linear transfer function $Z_l(j\omega)$ extracts one frequency or works as a discriminator. Thus, "filter hypothesis" holds true, so the describing function method can be applied here.

To find the angular oscillating frequency $\omega_0 = 2\pi f_0$ one needs to solve equation $Im\{Z_l(j\omega_0)\} = 0$ either analytically or numerically, since the nonlinear element is static.

Using the transfer function of a harmonically linearized nonlinear element $H_n(s, a, V_0)$ (where s is a Laplace variable, a is the first harmonic amplitude, V_0 is a DC component) one can find parameters of system oscillations according to Goldfarb and Kochenburger theory [3], [4]. According to this theory $H_n(s,a,V_0)$ acts as linear one at the stability threshold. So, one can write an oscillation condition (for the first harmonic) taking into account that a bode characteristic of the closed loop has to pass through a point $(1, j\cdot 0)$, or $H_n(s,a,V_0)Z_l(s) = -1$. In other words to find the amplitude an frequency of the oscillation one needs to find the intersection point between $Z_l(j\omega)$ and $-1/H_n(j\omega,a,V_0)$ in the complex plane.

If the nonlinear function is approximated with a polynom of the third order:

$$I_f = h_0 + h_1 \cdot u_{BE} + h_2 \cdot u_{BE}^2 + h_3 \cdot u_{BE}^3, \tag{3}$$

then the DC component of the base-emitter voltage (u_{BE_0}) can be found as a solution of the following equation:

$$k_3 u_{BE_0}^3 + k_2 u_{BE_0}^2 + k_1 u_{BE_0} + k_0 = 0, \tag{4}$$

where

$$k_3 = 2h_3, \quad k2 = (-12h_3 - 8h_2), \quad k_1 = \Big(-2h_1 - \frac{4}{Z_l(j\omega_0)} - 2h_2 - \frac{8}{3}\frac{h_2^2}{h_3} + \frac{1}{Z_l(0)}\Big)$$

$$k_0 = 2h_0 - \frac{4}{3}\frac{h_2 h_1}{h_3} - \frac{4}{3}\frac{h_2}{h_3 Z_l(j\omega_0)} - \frac{EH_z(0)}{Z_l(0)}.$$

And, the first harmonic amplitude is found as follows:

$$u_{be_1}^2 = -\frac{3h_3 u_{BE_0} + h_1 + 2h_2 u_{BE_0} + 1/Z_l(j\omega_0)}{\frac{3}{4}h_3}. \tag{5}$$

If more acurate solution is needed the higher harmonics may be found [3].

So, as we see all calculations of high-Q quartz oscillator may be performed in an analytical form for DC solution, first harmonic amplitude and oscillating frequency, though some other well-known approaches from the control system theory may be applyed here. The whole oscillator analysis does not require any simulator software, both simple SPICE or Harmonic Balance.

POWER DISSIPATED IN A CRYSTAL RESONATOR

The power dissipated by a quartz crystal resonator is a crucial parameter of the oscillator design. If the active power of the quartz is high enough then it evinces nonlinear phenomena and is a subject to fast ageing. For cryogenic crystal resonators this problem is more vital since the maximum possible level of dissipated power is much less at very low temperatures than at room temperature. Conversely if the active power is very small, than in real conditions the mechanical vibration cannot be excited. So, in the course of oscillator design and optimization, this parameter has to be taken into account.

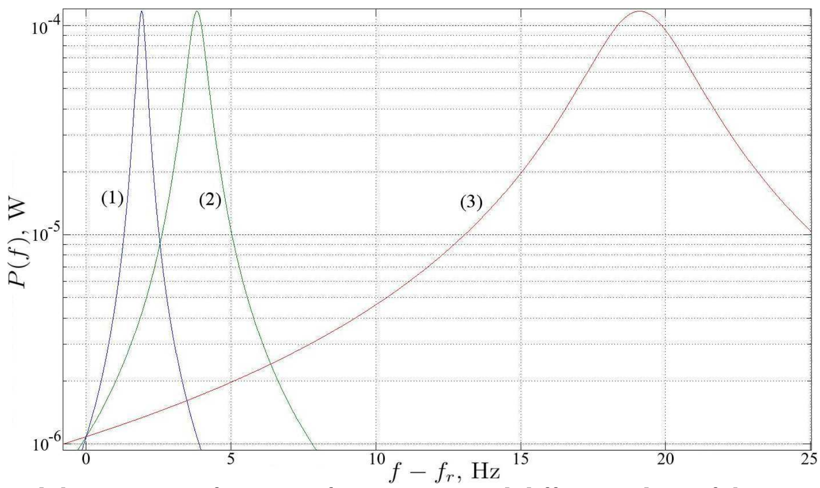

Fig. 5. $P(f)$ around the resonance frequency for $I_i = 1$mA and different values of the quartz crystal quality factor Q: (1) - 10^8, (2) - $5\cdot 10^7$, (3) - 10^7

For a given solution of the system described above one can find a current I_x and a voltage U_x of the quartz crystal (whose impedance is denoted as Z_x). In order to do that an oscillator can be represented as a two-port linear network from the nonlinear element to the quartz crystal itself. The nonlinear element has to be substituted for a current source I_i which has a frequency and an amplitude of the first harmonic corresponding to the given solution. After that a transfer function $H_i(j\omega)$ from the current I_i to current I_x has to be found with the help of the linear network analysis. Here the DC voltage source of the circuit has to be eliminated, since DC and first harmonic solutions are independent for linear networks and the quartz crystal does not have a DC current component.

The active power dissipated inside the crystal resonator can be expressed as:

$$P = Re(U_x(j\omega) \cdot I_x^*(j\omega)) = \left|H_i(j\omega)I_i(j\omega)\right|^2 Re(Z_x(j\omega)). \tag{6}$$

As an example Fig. 5 shows dependance $P(f)$ for a high-Q resonator (10^8) around a resonance frequency of about $f_r = 31.252$MHz for the current of nonlinear element equal to 1mA. Also, our numerical experiments have shown that for different values of Q and constant values of the resonance frequency, the active power dissipated inside a quartz crystal is the same, but in the same time oscillation frequency changes considerably.

OSCILLATOR QUALITY FACTOR

The oscillator quality factor is the principal parameter in the course of oscillator phase noise optimization. In order to achieve the highest possible frequency stability one needs to maximize this parameter.

The oscillator quality factor can be calculated from the transfer functions of the linearized oscillator open loop, i.e. its small signal model. This transfer function also can be found analytically. To do this the oscillator loop is divided into two parts (see Fig. 6): a resonant network and an amplifier with current transfer functions:

$$H_r(j\omega) = \frac{I_{o1}}{I_{i1}}, \quad H_a(j\omega) = \frac{I_{o2}}{I_{i2}}. \tag{7}$$

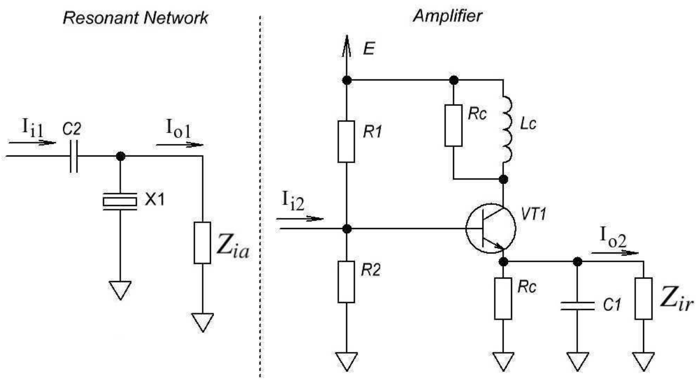

Fig. 6. Resonant network and amplifer with their loads

In order to find the transfer functions $H_r(j\omega)$ and $H_a(j\omega)$ one needs to find the load resistances Z_{ia} and Z_{ir}. For that, a closed oscillator loop can be represented (without lack of generality) as a infinite chain of subcircuits: amplifiers and resonant networks (see Fig. 6). If the resonant circuit is the first block of the chain, then the input impedance of this chain corresponds to the amplifier load, and vice versa.

The input impedance of the infinite chain Z_{ia} (Fig. 7) can be easily found using a simple statement: if the first periodic unit of the infinite chain with the input impedance Z_{ia} is removed, then the rest of the chain has the same input impedance Z_{ia}. This statement is obviously clear from the physical nature of the periodic infinite circuit. Also, this statement means that the rest of the infinite circuit can be substituted for the impedance Z_{ia}, in other words Z_{ia} is a function of Z_{ia}. This fact results in a second order algebraic equation of the following form:

$$AZ_{ia}^2 + BZ_{ia} + C = 0, \tag{8}$$

where A, B and C are parameters which depend on the circuit elements. The same type of equation may be found analytically for the load impedance Z_{ir}. This leads to analytical expressions of amplifier and resonant network transfer

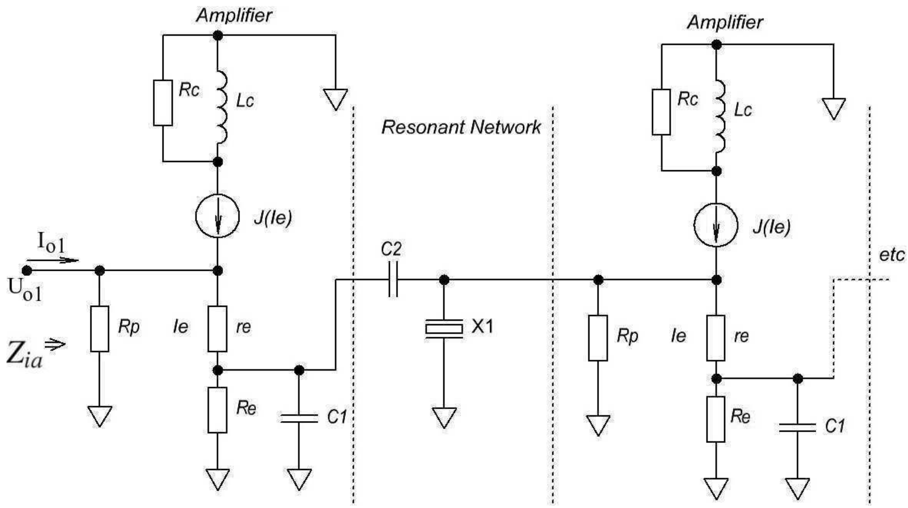

Fig. 7. Equivalent circuit for the load impedance of the resonant network (Z_{ia})

functions, which in our case are expressed as follows:

$$H_r(j\omega) = \frac{Z_x}{Z_x + Z_{ia}}, \quad H_a(j\omega) = \frac{R_p Z_e}{\big((1-\alpha)R_p + r_e\big)(Z_{ir} + Z_e) + Z_e Z_{ir}}, \tag{9}$$

where Z_e is the impedance of $C1$ and R_e parallel connection, R_p is the resistance of R_1 and R_2 connected in parallel, α and r_e are transsistor linearized parameters.

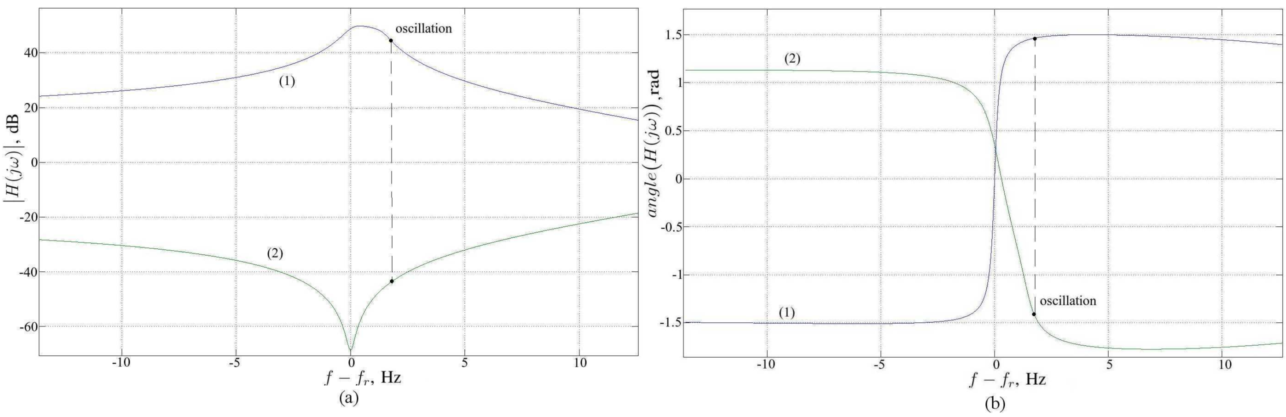

Fig. 8. (a) Amplitudes $\big|H_a(j\omega)\big|$ (1) and $\big|H_r(j\omega)\big|$ (2); (b) Phases $arg\big(H_a(j\omega)\big)$ (1) and $arg\big(H_r(j\omega)\big)$ (2)

Fig. 8 shows amplitude-frequency (a) and phase-frequency (b) characteristics of the resonant network and the amplifier respectively. Fig. 9 shows the dependence of $\frac{d\varphi}{df}$ versus frequency, which is sufficient to find the oscillator quality factor:

$$Q = -\frac{f_o}{2}\frac{d}{df} arg\big(H_r(2\pi f j) H_a(2\pi f j)\big)\Big|_{f=f_o} = -\frac{f_o}{2}\frac{d\varphi}{df}\Big|_{f=f_o}. \tag{10}$$

This expression also can be found analytically and is equivalent to results of SPICE AC analysis.

OSCILLATOR PARAMETRIC OPTIMIZATION

In introduction to this paper we have shown that optimization of very high-Q oscillators in the case of traditional simulation approaches is a manual tiring process. Attempts of optimization automation [5] meet three main problems: convergence problem (all types of analyses have to converge on each iteration step); optimization time problem (this process is extremely time-consuming); need for different types of analyses on each iteration step; limitation

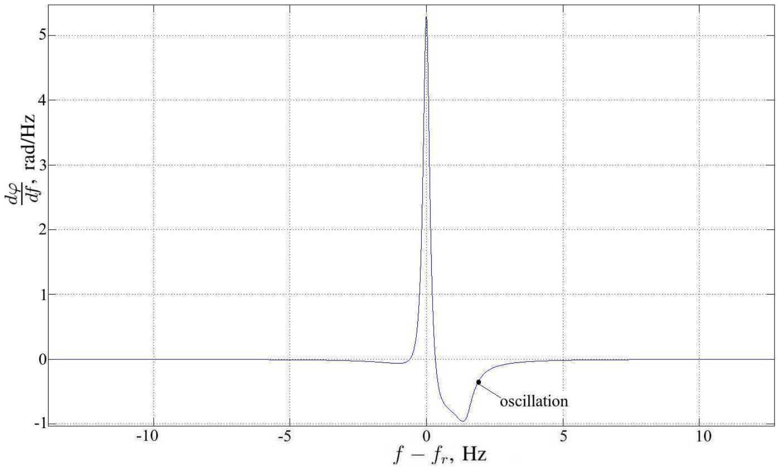

Fig. 9. Dependence of $\frac{d\varphi}{df}$ on frequecy

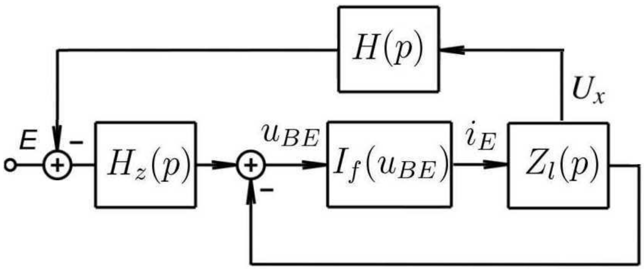

Fig. 10. Correction principle

of optimization algorithms posed by a simulating software. Contrary to that the proposed approach has no all this drawbacks. First, analytical methods have absolute convergence contrary to numerical. Even though some numerical techniques can be used to find the oscillating frequency in more sophisticated circuits, the convergence of such an algorithm may be rather high, since the task is to find the roots of a polynomial equation when an initial guess is very close to the solution. Second, the approach is very fast, since analytical expressions immediately give values of all parameters. Circuit transformations are performed just one time before the optimization process (an example of calculations on each iteration step is given in [5]). Third, method does not require any additional types of analysis with any of external simulating tools either SPICE or HB. So, it well suits for home-made software realization and any appropriate optimization algorithm may be used with the presented approach.

For oscillator optimization, derivative-free methods are the most suitable. Amoung them we can highlight the Nelder-Mead method, genetic algorithms and simulated annealing, which have been successfully used for optimization of high-order electro-mechanical systems [7].

OSCILLATOR NOISE CORRECTION

In the considered case not only parametric, but also structural optimisation is possible, when the system structure is changed by an additional feedback. This feedback can result in better phase noise reduction among other effects. Though no structural changes are introduced into the main oscillating loop the dynamical behaviour of the system can be changed significantly. The idea of such a correction becomes clear considering Fig. 10. The main oscillating loop of the system is positive (since $Z_l(j\omega_0)$ is real and negative). The second negative loop makes a signal path from a measurable system variale of linear part $Z_l(j\omega)$ to the system input variable E, i.e. power source voltage. So, in a corrected oscillator, E is no more constant as in the usual case.

We have demonstrated this effect with a real prototype, which is a simple room-temperature quartz crystal Colpitts-type oscillator (our prototype does not have a voltage regulator and an output isolation amplifier. The temperature control of the crystal is not adjusted at its turn-over point). Its measured Allan deviation with and without additional feedback is shown in Fig. 11. The quartz crystal voltage U_x is chosen here as a reference variable for the second loop.

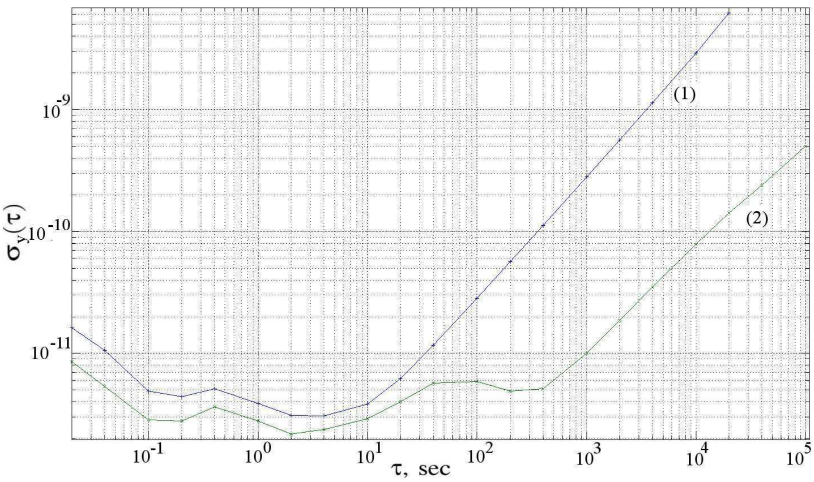

Fig. 11. $\sigma_y(\tau)$ for the oscillator measured against an H-maser: (1) without correction, (2) with correction

Though this oscillator has not been optimized yet the simple addition feedback demonstrates considerable noise reduction. Further consideration of this effect is a subject of another work.

CONCLUSION

This work presents a technique, which can be successfully used for optimization of high-*Q* oscillators in terms of phase noise. This technique is based on analytical calculations and helps to find all important oscillator parameters, such as oscillation frequency, DC solution, first harmonic amplitude, circuit quality factor. The oscillation frequency can be made analytically with the help of some numerical techniques. The advantages of the approach make it easy to implement the oscillator automatic optimization without any SPICE or HB simulators. The presented transforms help to represent the oscillator in the system form which is widely used in control system theory and consequently use its methods. Besides parametric optimization the given transform shows the idea of structural optimization with the help of an external feedback. Experimental results show the effectiveness of such a correction.

This work is supported by Conseil Régional de Franche-Comté (Convention No. 2008C_16215). The authors wish to thank them for this grant and financial support as well as the staff of the Time and Frequency Department of FEMTO-ST Institute, Besançon, France for their help and fruitful discussions.

REFERENCES

[1] S. Galliou, J. Imbaud, R. Bourquin, N. Bazin, P. Abbé, “Outstanding quality factors of bulk acoustic wave resonators at cryogenic temperature,” *Proc. 22nd EFTF*, Toulouse, France, May 2008.

[2] S. Galliou, J. Imbaud, R. Bourquin, N. Bazin, P. Abbé, “Quartz crystal resonators exhibiting extremely high Q -factors at cryogenic temperatures,” *Electronics Letters*, Vol. 44, no 14, p. 889-890, July 2008.

[3] V.A. Besekersky, E.P. Popov, *Automatic Control System Theory*, Professiya, Moscow, 2004.

[4] W.S. Levine, *Control System Fundamentals*, CRC Press, New York, 1999.

[5] J. Imbaud, S. Galliou, P. Abbé, “Measurement of First Langatate Oscillators Improved by an Original Simulating Method,” *Proc. 22nd EFTF*, Toulouse, France, May 2008.

[6] M. Addouche, R. Brendel, D. Gillet, N. Ratier, F. Lardet-Vieudrin, J. Delporte, “Modeling of quartz crystal oscillator by using nonlinear dipolar method,” *IEEE Trans. UFFC*, pp. 487–495, May 2003.

[7] G.C. Onwubolu, B.V. Babu, *New Optimization Techniques in Engineering*, Springer, Berlin, 2004.

[8] M. Goryachev, S. Galliou, Ph. Abbé, “Cryogenic Transistor Measurement and Modeling for Engineering Applications”. Cryogenics, Elsevier, in press.